

# Decoherence without decoherence<sup>☆</sup>

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## Abstract

It has been claimed that decoherence of open quantum systems explains the tendency of macroscopic systems to exhibit quasiclassical behavior. We show that *quasiclassicality* is in fact an unremarkable property, characterizing generic subsystems of environments even in the absence of dynamical decoherence. It is suggested that decoherence is best regarded as explaining the persistence of true classicality, rather than the emergence, rather than the emergence of quasiclassicality.

*Key words:* decoherence, classical, quasiclassical, emergence

*PACS:* 03.65.Ta, 03.65.Yz, 05.30.Ch, 98.80.Qc

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## 1. Introduction

Over the last four decades [1], the study of decoherence has begun to shed light on the effects of the interaction of open quantum systems with their environments. It has been shown that, for some interesting model systems, certain pure states, sometimes called *pointer states* [2][3][4] survive interaction with the environment without losing their coherence or purity. This in turn implies that superpositions of these states lose coherence in such a way that the result is an incoherent, improper *mixture* of such states which is approximately stable. The fact that macroscopic subsystems interacting with an appropriate environment can be seen to exhibit decoherence in a preferred basis, along with the fact that the basis in question often corresponds to a paradigmatically classical observable such as position, has led to claims that “the classical structure of phase space emerges from the quantum Hilbert space in the appropriate limit” [5]; that “the appearance of classicality is therefore grounded in the structure of the physical laws governing the system-system environment interactions” [6]; and that “there are strong signs that the transition [from quantum to classical] can be understood as something that emerges quite naturally and inevitably from quantum theory” [7]. Other, similar claims lie ready to hand [8][9]. Thus classicality is supposed not to be endemic to quantum theory, but to emerge naturally via certain natural interactions when sufficiently macroscopic objects interact with their environment.

Criticisms of the decoherence program (see e.g. [10]) have to date focused largely on the fact that the phenomenon in question is only known to occur for certain model Hamiltonians. If the properties of these Hamiltonians are not generic, then the possibility of a general explanation of

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<sup>☆</sup>Thanks to Miles Blencowe, Cliff Burgess, Paul Davies, Jim Hartle, Lee Smolin, Max Schlosshauer, Dieter Zeh, and Wojciech Zurek for comments on an early draft, and to Robin Blume-Kohout, Amit Hagar, Adrian Kent, Owen Maroney and Jos Uffink for interesting and enjoyable discussions.

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the emergence of classicality is undermined. The concern of this paper is, however, orthogonal. Rather than contesting the generality of the models under consideration, we will focus on a single canonical model – the central-spin model – and show that the features which the central spin attains via the decoherence process are equally features of the subsystems of its environment, implying that there is nothing especially interesting about the quasiclassicality which is supposed to characterize the central spin in the wake of decoherence. The truly classical states, in the sense considered here, will be shown to be the pointer states themselves, states which by definition do *not* undergo decoherence.

## 2. Decoherence and classicality

The process of decoherence works roughly as follows. Consider a subsystem  $S$  with (pure) state  $\psi^S$  interacting with an environment  $E$  with state  $\psi^E$ . If the subsystem is sufficiently macroscopic, and if the Hamiltonian governing the combined evolution of subsystem and environment is appropriate, then the environment as a whole acts as a kind of measuring device, in that the effective state of the environment (given by its reduced density matrix) will reliably become correlated with certain subsystem observables. Which properties of the system are “measured” by the environment – which observables (if any) become nontrivially correlated – will depend on the Hamiltonian, including the self-Hamiltonians of system and of environment [11][12]. Eigenstates of the subsystem observables in question, the pointer states, will be stable or approximately stable under such measurement-like interactions, while arbitrary *superpositions* of pointer states will evolve into improper *mixtures* of those states as a result of the environment’s correlation with the pointer observable. The tendency for the reduced density matrix of the subsystem to be driven into a small subset of the available states by the environment is called *einselection*, short for *environment induced superselection* [3][5]. Decoherence, then, refers to the process by which pure states lose their coherence, and more particularly to a process which favors a particular basis.

What does this all have to do with classicality? The driving idea is that classicality has to do with the stability over time of typical classical observables such as position, momentum, or energy.<sup>1</sup> Thus if we have an interaction which picks out position eigenstates of a certain subsystem as pointer states, then the subsystem may be said behave classically in case it is in one of the pointer states, since it can be predicted to have a definite trajectory in space. Indeed, this is quite similar to the “criterion of reality” stipulated by Einstein, Podolsky and Rosen [14], whereby a property is attributable to a system if it can be said to possess that property with certainty.

Associating the unambiguous possession a particular property with classicality is unexceptionable, but the claims in the decoherence literature typically associate a more general kind of classicality, sometimes distinguished as “quasiclassicality” [6] with a larger class of subsystems, those described by improper *mixtures* of pointer states. The idea behind calling such states “quasiclassical” seems to be that they behave like ensembles of classical subsystems with respect to the observable of interest. Interference effects in particular are wiped out. For example, consider a two-slit interference experiment with electrons. The introduction of an appropriate environment such as dust or visible light has the effect of inducing decoherence and destroying the interference pattern, and the electrons furthermore behave as if they are members of a classical ensemble of

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<sup>1</sup>See [13] for a careful and clear discussion of several different senses of classicality that might be brought to bear in this context.

particles, some of which emerge from one slit and some from the other. Thus the observed behavior of the electrons has affinities with the behavior of particles described by classical mechanics, and is said to be quasiclassical. The “quasi” is in place because the mixture refers, not to actual ensemble, but to a single system, having been obtained by tracing out the environmental degrees of freedom with which it is entangled.

Perhaps the simplest example of a system which exhibits decoherence to a preferred basis is the central spin model. Here we find that the central spin, initially in a pure state, evolves into an incoherent mixture of  $z$  eigenstates, unless, that is, the initial state is itself a  $z$  eigenstate. What we will show is that an arbitrary state of the spin’s environment – an arbitrary state of any other spin – is *also* an incoherent mixture of  $z$  eigenstates. Thus what is offered as a distinctive feature of the central spin turns out to be a generic feature of an arbitrary subsystem. Thus the supposed property of quasiclassicality is better thought of as a generic feature of quantum states, one which has to do with the fact that on any reasonable measure, most subsystems are massively entangled with other subsystems, and thus “already-decohered”.

### 2.1. Example: Central spin model

Consider for example the so-called central spin model [3] in which one contemplates a system consisting of  $N + 1$  two-level systems,  $N$  of which are coupled to a central spin  $S$  via the Hamiltonian

$$\hat{H} = \frac{1}{2} \hat{\sigma}_z \otimes \left( \sum_{i=1}^N g_i \hat{\sigma}_z^{(i)} \bigotimes_{i' \neq i} \hat{I}_{i'} \right)$$

where  $\hat{I}_i$  is the identity operator for the  $i$ ’th system. (Here there is no macroscopic/microscopic distinction; rather, the distinctive dynamical role of the central spin singles it out as special.) An initial pure state of the form

$$\psi = \alpha | +z \rangle | E_0 \rangle + \beta | -z \rangle | E_0 \rangle \quad (1)$$

will, via the unitary evolution  $U(t) = e^{-i\hat{H}t}$  generated by this Hamiltonian, evolve toward an entangled state  $\psi(t) = \alpha | +z \rangle | E_+(t) \rangle + \beta | -z \rangle | E_-(t) \rangle$ . After a sufficient amount of time  $t_d$  has passed,  $\langle E_+ | E_- \rangle \approx 0$ , and the reduced density matrix of the central spin will be well-approximated by

$$\rho^S = \alpha^2 | +z \rangle \langle +z | + \beta^2 | -z \rangle \langle -z |. \quad (2)$$

One can represent this evolution on the Bloch sphere as the evolution of initially pure states of the central qubit (the surface of the sphere), evolving, modulo extremely unlikely Poincare-type fluctuations, toward a narrow ellipse along the  $z$  axis:

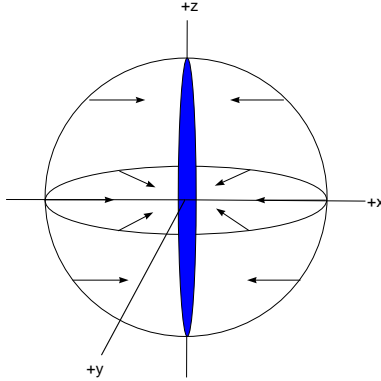


FIG 1 (color online): Evolution of the central spin. Points away from the  $z$  axis move toward it along  $z = \text{constant}$  surfaces.

Though it is no surprise that a subsystem should lose coherence upon interaction with its environment, and thus move away from the surface of the Bloch sphere, we have here in addition the phenomenon of einselection, in which the loss of coherence occurs in a preferred direction. In particular, the loss of purity is proportional to the angle with the  $z$  axis, with the pointer states  $|+z\rangle$  and  $|-z\rangle$  suffering no loss whatsoever.

### 3. Properties of the environment

The superselection of a preferred set of states corresponding to eigenstates or near-eigenstates of typical classical observables is, on the face of it, an interesting phenomenon suggestive of an emergent quasiclassicality. What we will now show is that in fact, typical quantum subsystems are in many cases already in such states – they are *already decohered*.

Let us proceed by looking at the central spin example in more detail. The initial state of the environment, denoted  $|E_0\rangle$  above, is a pure state about which we have no information *a priori*. (Note that, in contrast to our earlier treatment [15], we are granting the assumption that the system and the environment are initially in a product state.) A “proper” mixture expressing our ignorance is represented by the density matrix

$$\Omega_E = \sum_{i=1}^d \frac{1}{d} |\psi_i\rangle \langle \psi_i|$$

where  $d = 2^n$  is the dimension of the Hilbert space of the  $n$  qubits that make up the environment, and where the  $|\psi_i\rangle$  are orthonormal basis vectors for this space. This corresponds to an unbiased probability distribution with respect to the unitarily invariant Haar measure, reflecting our complete ignorance as to which pure state the system is in at the outset. It is “maximally-mixed”, exhibiting random behavior with respect to any choice of observable, and its von Neumann entropy is therefore maximal.

Let us now inquire as to the description of an arbitrarily chosen subsystem  $e_1$  of the environment  $E$ , where  $H_E = H_{e_1} \otimes \dots \otimes H_{e_n}$ , corresponding to the  $n$  spins which make up the environment. Given the mixture  $\Omega_E$ , the effective state of  $e_1$  is the reduced state  $\Omega_{e_1} = \text{Tr}_{\tilde{E}} \Omega_E$ , where  $\tilde{E}$  refers to the rest of the environment ( $H_{\tilde{E}} = H_{e_2} \otimes \dots \otimes H_{e_n}$ ). Since  $\Omega_e$  is a multiple of the identity, so too is  $\Omega_{e_1}$ ; both are maximally mixed states.

Suppose, now, the environment starts out in some particular unknown pure state  $\rho_E$ . We can think of this, if we like, as obtained by random sampling from the distribution  $\Omega_E$ . Thus a particular environmental spin  $e_1$  will be described by the density matrix  $\rho_{e_1} = \text{Tr}_{\tilde{E}} \rho_E$ . It may be in a pure state, or a mixed state, but it is more likely to be in a mixed state. In fact, it has been shown [16][17] that the state of the subsystem  $\rho_{e_1}$  will be almost indistinguishable from the state  $\Omega_{e_1}$ . More specifically, the average value of the “trace distance”  $D(\rho, \Omega) := \frac{1}{2} \text{Tr}(|\rho - \Omega|)$  [18][19] between the two states is bounded by

$$0 < \langle D(\rho_{e_1}, \Omega_{\tilde{E}}) \rangle \leq \frac{d_{e_1}}{2} \sqrt{\frac{1}{d_{\tilde{E}}}} \quad (3)$$

(where  $d_{e_1}$  and  $d_{\tilde{E}}$  are respectively the dimensions of  $e_1$  and  $\tilde{E}$ ), so that  $D(\rho_{e_1}, \Omega_{e_1}) \approx 0$  for almost all states  $\rho_{e_1}$ . In other words, if one takes an arbitrary pure state of the environment, then an arbitrary small subsystem of the environment will be very well approximated by a maximally mixed state. *A fortiori*, in the case of the central spin model, an environmental spin will, with overwhelming likelihood, live in the superselection sector. This is a purely kinematic fact, involving no dynamics, no loss of quantum coherence. One might, in the manner of John Wheeler, call this *decoherence without decoherence*.

It is also salient to note that this feature of subsystems survives the decohering interaction, for the simple reason that the ensemble from which it is drawn – in this case  $\Omega_{e_1}$  – remains maximally mixed throughout the evolution. Thus environmental subsystems will, with overwhelming probability, reside in the superselection sector which is supposed to be characteristic of “classical” systems, and will remain there indefinitely.

#### 4. Classicality revisited

What we have shown is straightforward to the point of being obvious, in retrospect. In the model under consideration, the central spin will evolve from an initially pure state into a state which resides in the ostensibly quasiclassical superselection sector, while a random environmental spin will almost *always* be found in this superselection sector. This implies that *quasiclassicality* is not a particularly interesting property.

What *is* interesting, on the other hand, is classicality *simpliciter*, in which the quantum state assigns the system a definite value over time. This is of course characteristic of the pointer states, but *not* of superpositions thereof, and not of environmental spins. What “decoherence” does to pointer states is in fact to maintain their classicality by precluding a loss of coherence. Decoherence does not explain the *emergence* of classicality, but its *persistence*. It does so by preventing the loss of coherence in the basis of one or more observables. The emergence of classicality, on the other hand, appears to await a resolution of the so-called “measurement problem” – only when physical properties take on definite values does one have something resembling a classical world.

#### References

- [1] H. Zeh, On the interpretation of measurement in quantum theory, Foundations of Physics 1 (1970) 69–76.
- [2] W. Zurek, Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?, Phys. Rev. D24 (1981) 1516–1525.

- [3] W. Zurek, Environment-induced superselection rules, *Phys. Rev. D* 26 (1982) 1862.
- [4] M. Simonius, Spontaneous symmetry breaking and blocking of metastable states, *Phys. Rev. Lett.* 40 (1978) 980–983.
- [5] W. Zurek, Decoherence, einselection, and the quantum origins of the classical, *Rev. Mod. Phys.* 75 (2003) 715.
- [6] M. Schlosshauer, *Decoherence and the Quantum-to-Classical Transition*, Springer, Berlin, 2007.
- [7] P. Ball, Quantum all the way, *Nature* 453 (2008) 22.
- [8] W. Zurek, Quantum measurements and the environment-induced transition from quantum to classical, in: A. Ashtekar, J. Stachel (Eds.), *Conceptual Problems of Quantum Gravity*, Birkhäuser, Boston, 43–66, 1991.
- [9] E. Joos, H. Zeh, C. Kiefer, D. Giulini, J. Kupsch, I.-O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory*, Springer-Verlag, Berlin, 2nd edn., 2003.
- [10] P. Stamp, The Decoherence Puzzle, *Stud. Hist. Phil. Mod. Phys.* 37 (2006) 467–497.
- [11] L. Tessieri, J. Wilkie, Decoherence in a spin–spin-bath model with environmental self-interaction *ArXiv:quant-ph/0209079v1*.
- [12] C. Dawson, A. Hines, R. McKenzie, G. Milburn, Entanglement Sharing and Decoherence in the Spin-Bath, *Phys. Rev. A* 71 (2005) 052321.
- [13] H. Janssen, *Reconstructing Reality: Environment-Induced Decoherence, the Measurement Problem, and the Emergence of Definiteness in Quantum Mechanics* Draft, available at <http://philsci-archive.pitt.edu/archive/00004224/>.
- [14] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of reality be considered complete?, *Phys. Rev.* 47 (1935) 777–780.
- [15] S. Weinstein, Decoherence and the (non)emergence of classicality *ArXiv:0807.3376v2*.
- [16] D. Page, Average Entropy of a Subsystem, *Phys. Rev. Lett.* 71 (1993) 1291–1294.
- [17] S. Popescu, A. J. Short, A. Winter, Entanglement and the foundations of statistical mechanics, *Nature Physics* 2 (2006) 754–758.
- [18] C. Fuchs, *Distinguishability and Accessible Information in Quantum Theory* .
- [19] M. Nielsen, I. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, 2000.